A derivative

3.3 #27) $y=\left(\frac{3x+2}{x}\right)\left(x^{-5}+1\right)$. Find $dy/dx\big|_{x=1}$. In other words, find the derivative (function) and then evaluate it at an $x$ – value of 1.

We'll use the product rule … but we still have choices. Let's organize our functions ☺

$f(x)=\left(\frac{3x+2}{x}\right);\ g(x)=\left(x^{-5}+1\right)$

$f'(x)=\frac{(3)(x)-(3x+2)(1)}{x^2}=\frac{-2}{x^2};$

$g'(x)=-5x^{-6}$

We used the quotient rule to evaluate *f'* and the power rule to evaluate *g'*.

Now to the product rule, and our discovery of the derivative.

$\frac{dy}{dx}=f'g+fg'=\left(\frac{-2}{x^2}\right)\left(x^{-5}+1\right)+\left(\frac{3x+2}{x}\right)\left(-5x^{-6}\right)$. In general we aren't going to simplify answers in AP Calc unless there is a compelling reason. Here there is absolutely NO compelling reason since what I'm going to do next is just substitute 1 in for $x$.

So $f'(1)=\ f'(1)=\left(\frac{-2}{1}\right)(1+1)+\left(\frac{5}{1}\right)(-5)=-4-25=-29$

If you wanted to avoid the quotient rule for *f'*, we could notice that $\frac{3x+2}{x}=\frac{3x}{x}+\frac{2}{x}=3+\frac{2}{x}=3+2x^{-1}$.
And calculating the derivative of that requires only the power rule.

Hope this helps!